

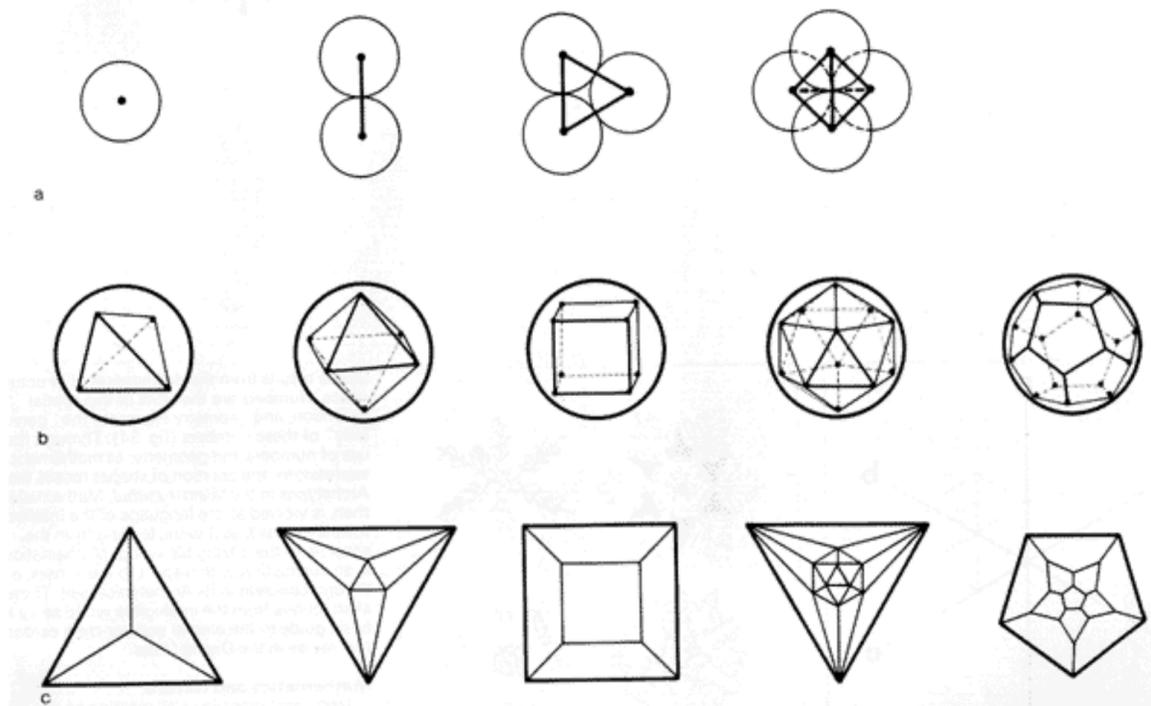
The Mathematics of Proportion

One, as creator, commences with the point, moves through two, as the line, whose action as a radius generates the sphere. The sphere is the most evident symbol of Unity and its division by inscribed regular polygons constitutes the basis of all traditional laws of proportion.

There are only five regular polygons which can be inscribed in a sphere (fig. 36). Known as the "Platonic bodies," they have been described by Al-Biruni as follows: "These five are related by resemblance to the four elements and the sphere (Universe). With regard to the five, they are, first, the cube (hexahedron) bounded by six squares called 'earthy'; second, the icosahedron, by twenty equilateral triangles, the 'watery' one; third, the octahedron, by eight equilateral triangles, the 'airy' body; fourth, the tetrahedron, by four equilateral triangles, the 'fiery' body; and fifth, the dodecahedron by twelve pentagons," the symbol of the universe as a whole.

Mathematics of Proportion: Platonic Bodies

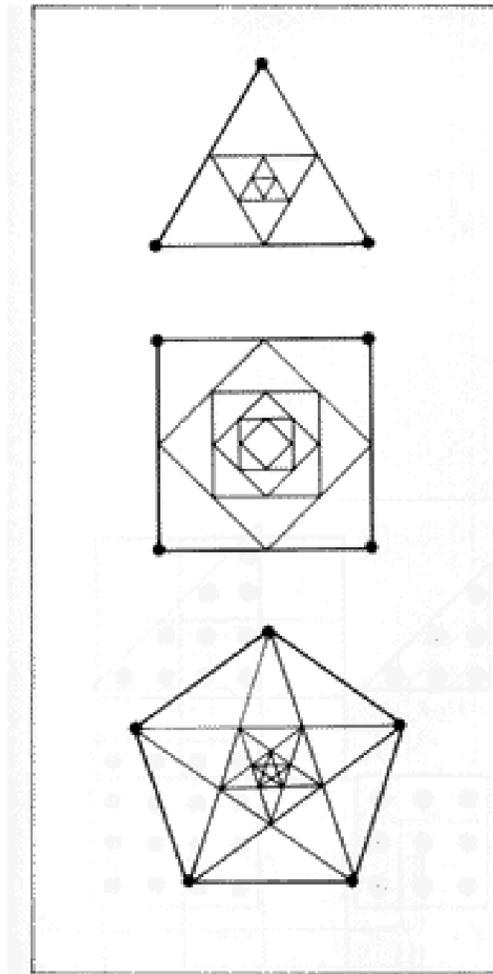
(a) The essential movements through the dimensions of space begin with Unity. The morphic point moves through 2, the line; 3, the triangular plane; and 4, the most simple and primary solid, the tetrahedron.



(b) "There is moreover no better symbol in the visual order of the internal complexity of Unity - of the passage from Indivisible Unity to 'Unity in multiplicity' or 'multiplicity in Unity' - than the series of the regular geometric figures contained within a sphere" (T. Burckhardt, "Perennial Values in Islamic Art," *Studies in Comparative Religion* 1: 3). The realization of these regular polyhedra known as the platonic bodies are attributed to Pythagorean thought which, it is believed, strongly influenced the cosmology of Plato as manifested in the *Timaeus*.

(c) Planar graph representations of the five regular solids.

Taking the dodecahedron, one obtains the pentagon as the planer surface construction. When its five vertices are connected, the pentacle, or fivepointed star, is generated (fig. 37). Further linking of points of the pentacle results in harmonically diminishing pentagons and pentacles (fig. 38).



37. The triangle, square, and pentagon related to the first three Platonic bodies generate diminishing harmonic patterns. Diagram (a) indicates the rootthree proportional system of triangles; (b) the roottwo proportional system of squares exhibits successive squares whose edges diminish by the square root proportion of the previous edges; (c) indicates a series of diminishing pentagrams harmonically related through the golden mean ratio

38. *Pythagoras's Lute* This diagram shows a progression of pentacles..... the diagram is self-illustrating in terms of the relationship of the stars to each other. The harmonic nature of the progression is expressed in the fact that the relationship between 1 (bottom right) and a is a golden mean proportion to the length between 1 and 2. Also the distance from a to 2 is equal to that between 2 and 3. This ratio is based on the 'extreme mean' proportion, i.e. in any given line, say from a to b, there is a position, x, on the line so that the distance from a to x is greater than from x to b and in the same proportion as the whole line ab is larger than the distance from a to x. This is expressed as a fraction,

$$\frac{1+\sqrt{5}}{2} = 1.55$$

and the result, like the other proportional fractions, is an irrational number, i.e. it will not resolve into a whole number. Pythagoras is credited with having discovered the proportional lengths of taut string to musical sound, and these relationships are associated with length proportions." (K. Critchlow, *Order in Space*, p. 84.)

